TURBULENT FRICTION AND HEAT EXCHANGE IN STREAM SWIRLING IN A PIPE

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Much interest is devoted to the investigation of processes of turbulent transfer of heat and momentum in swirled streams in pipes. A number of experimental [1-12] and theoretical [12-19] papers can be named where these phenomena were analyzed. A detailed comparison of the available calculated and experimental data, however, indicates their considerable disagreement.

Swirled streams are distinguished by great variety even in the qualitative flow pattern, which is determined primarily by the geometrical and flow-rate characteristics. The diversity of ways of swirling a stream – by vane swirlers at the channel entrance, a tangential supply of gas, ribbon and screw swirlers, etc. – considerably complicates the analysis and generalization of experiments. The method of generalizing test data as a function of the geometrical and flow-rate conditions at the entrance of the specific device being studied has become very popular. A detailed elucidation of questions connected with heat exchange in channels with different methods of stream swirling is given in [18].

A common result of all the research is the conclusion that the various methods of stream swirling have an intensifying action on heat and mass transfer and surface friction. But there is still no single point of view on the factors causing the intensification of transfer processes in stream swirling. Thus, in a number of papers [13, 16, 17, 19] the increase in heat transfer and friction is explained by "geometrical" factors alone, i.e., through an increase in the total velocity and the length of a streamline. At the same time, according to the data of [12, 14, 15, 18, 20], the influence of the mass forces due to the curvature of the streamlines on the structure of the turbulence and the transfer processes may be considerable. In the present paper we solve the problem of heat and mass exchange and friction in the initial section of a pipe with stream swirling with allowance for the above-indicated phenomena.

1. Heat and Mass Exchange and Friction in a Swirled Stream Without Allowance for the Action of Mass Forces on Turbulence

The integral equations for the longitudinal component of the momentum and the heat of a swirled stream can be obtained from the corresponding system of differential equations of motion, energy, and continuity of the boundary layer in a cylindrical coordinate system [21]. For axisymmetric flow in the initial section of an impermeable round pipe with a sufficiently thin boundary layer, $\delta/R \ll 1$, when the flow can be considered as zero-gradient ($\partial u_{x0}/\partial x = 0$), the integral equations have the form

$$\frac{d\,\mathrm{Re}_x^{**}}{d\bar{x}} = \frac{c_{fx}}{2}\mathrm{Re}_D = \Psi\frac{c_{fx0}}{2}\mathrm{Re}_D; \tag{1.1}$$

$$\frac{d\,\mathrm{Re}_T^{**}}{d\bar{x}} + \frac{\mathrm{Re}_T^{**}}{\Delta T}\frac{d(\Delta T)}{d\bar{x}} = \mathrm{St}\,\mathrm{Re}_D = \Psi_T\,\mathrm{St}_0\,\mathrm{Re}_D, \tag{1.2}$$

where $\mathrm{Re}_x^{**} = \rho_0 u_{x0}\delta_x^{**}/\mu_0$ and $\mathrm{Re}_T^{**} = \rho_0 u_{x0}\delta_T^{**}/\mu_0$ are the Reynolds numbers constructed from the momentum thickness in the longitudinal direction and the energy thickness, respectively,

$$\delta_x^{**} = \int\limits_{R-\delta}^{R}\frac{\rho u_x}{\rho_0 u_{x0}}\left(1-\frac{u_x}{u_{x0}}\right)\frac{r}{R}\,dr,\quad \delta_T^{**} = \int\limits_{R-\delta}^{R}\frac{\rho u_x}{\rho_0 u_{x0}}\left(1-\frac{T_w-T}{T_w-T_0}\right)\frac{r}{R}\,dr; \tag{1.3}$$

$\bar{x} = x/D$; $\mathrm{Re}_D = \rho_0 u_{x0}D/\mu$; $\Delta T = T_w - T_0$; R, δ, and u_{x0} are the radius of the channel, the thickness of the boundary layer, and the longitudinal velocity at its outer boundary.

The integral equations of momentum (1.1) and energy (1.2) have the same form as for unswirled flow [22], while the influence of stream swirling on the characteristics of the

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transfer processes is felt through the relative coefficients of friction $\Psi = (c_{fx}/c_{f0})_{\mathrm{Re}^{**}}$ and heat exchange $\Psi_T = (\mathrm{St}/\mathrm{St}_0)_{\mathrm{Re}_T^{**}}$. Here $c_{fx}/2 = \tau_x/\rho_0 u_{x0}^2$ and $\mathrm{St} = q_w/\rho_0 u_{x0} c_p \Delta T$ are the coefficients of friction and heat exchange in the swirled stream while $c_{f0}/2$ and St_0 are the analogous parameters but in the absence of swirling.

To solve the integral equations (1.1) and (1.2), we must obtain expressions for the laws of friction and heat exchange in a swirled stream. For this purpose we use the Prandtl hypothesis for a turbulent three-dimensional boundary layer; the connection between the shear stresses and the averaged parameters of the flow is written in the form

$$\tau_\Sigma = \rho l^2 (\partial u_\Sigma / \partial r)^2, \tag{1.4}$$

where $\tau_\Sigma = \sqrt{\tau_x^2 + \tau_\varphi^2}$, $u_\Sigma = \sqrt{u_x^2 + u_\varphi^2}$ are the total shear stresses and the total velocity. Using the condition of constancy of the angle of stream swirling over the thickness of the boundary layer, $\varphi = \arctan(u_{\varphi 0}/u_{x0}) = \mathrm{const}$ [18], we write the longitudinal component of the shear stress as

$$\tau_x = \tau_\Sigma \cos\varphi = \rho l^2 (\partial u_x/\partial r)^2/\cos\varphi. \tag{1.5}$$

For the turbulent heat flux we can obtain an expression similar in structure,

$$q = c_p \rho l l_T \frac{\partial u_\Sigma}{\partial r}\frac{\partial T}{\partial r} = c_p \rho l l_T \frac{\partial u_x}{\partial r}\frac{\partial T}{\partial r}\frac{1}{\cos\varphi}. \tag{1.6}$$

In accordance with the asymptotic theory of a turbulent boundary layer [22], we obtain the limiting relative law of friction from (1.5)

$$\Psi = \left(\frac{c_{fx}}{c_{f0}}\right)_{\mathrm{Re}_\Sigma^{**}} = \frac{1}{\cos\varphi}\left[\int_0^1 \sqrt{\frac{\rho\tilde{\tau}_0}{\rho_0\tilde{\tau}}}\,\frac{l}{l_0}\,d\omega_x\right]^2. \tag{1.7}$$

For the boundary layer of a swirled stream we can assume [18] that the distribution of the relative values of the shear stress has the same character as in the absence of swirling, $\tilde{\tau}/\tilde{\tau}_0 = (\tau/\tau_w)/(\tau/\tau_w)_0 = 1$. The density distribution will also coincide with the dependence for flow at a plate [22],

$$\rho/\rho_0 = \psi + (1-\psi)\omega_x.$$

Here ρ and ρ_0 are the density at the point under consideration and in the core of the flow; $\psi = T_w/T_0$ is the nonisothermicity factor; $\omega_0 = u_x/u_{x0}$.

In Eq. (1.7) the influence of the curvature of the streamlines on the turbulent characteristics (see, e.g., [20]) is taken into account through the ratio of the mixing lengths for swirled and unswirled flows, ℓ/ℓ_0.

First let us analyze the influence of stream swirling only through the average characteristics of the flow, without allowance for the action of mass forces on the turbulence ($\ell = \ell_0$). And for simplicity, we consider the case of quasi-isothermal flow ($\rho \approx \rho_0$). Under these conditions, Eq. (1.7) takes the form

$$\Psi = \Psi_\varphi = (c_{fx}/c_{fx0})_{\mathrm{Re}_\Sigma^{**}} \doteq (\mathrm{St}/\mathrm{St}_0)_{\mathrm{Re}_{\Sigma T}^{**}} = \frac{1}{\cos\varphi}. \tag{1.8}$$

The comparison of the exchange coefficients in the swirled stream under consideration and in a channel without swirling in (1.8) is made for the Reynolds number calculated from the total velocity at the outer limit of the boundary layer: $\mathrm{Re}_\Sigma^{**} = \rho_0 u_{\Sigma 0}\delta_\Sigma^{**}/\mu$. The quantity $\delta_\Sigma^{**} = \int_{R-\delta}^{R} \frac{\rho u_\Sigma}{\rho_0 u_{\Sigma 0}}\left(1 - \frac{u_\Sigma}{u_{\Sigma 0}}\right)\frac{r}{R}\,dr$ is the momentum thickness in the direction of the total velocity vector; with similarity of the velocity profiles in the boundary layer, $u_x/u_{x0} = u_\varphi/u_{\varphi 0} = u_\Sigma/u_{\Sigma 0}$, its value equals the momentum thickness in the longitudinal direction, $\delta_\Sigma^{**} = \delta_x^{**}$. Then, using the friction law $c_{f0}/2 = (B/2)\mathrm{Re}_\Sigma^{**-m}$ under standard conditions [22] and considering that $u_\Sigma = u_x/\cos\varphi$, from (1.8) we obtain expressions for the coefficient of friction in the longitudinal direction

$$\frac{c_{fx}}{2} = \frac{B}{2}\left(\frac{\rho_0 u_{\Sigma 0}\delta_\Sigma^{**}}{\mu}\right)^{-m}\frac{1}{\cos\varphi} = \frac{B}{2}\left(\frac{\rho_0 u_{x0}\delta_x^{**}}{\mu_0}\right)^{-m}(\cos\varphi)^{m-1} \tag{1.9}$$

and the coefficient of heat exchange

$$\mathrm{St} = \frac{B}{2}\left(\frac{\rho_0 u_{x0}\delta_T^{**}}{\mu_0}\right)^{-m} \mathrm{Pr}^{(m-1)}(\cos\varphi)^{m-1}. \quad (1.10)$$

For Re** < 10^4 B/2 = 0.0128 we have m = 0.25. If we assume that the turbulent boundary layer is formed right from the entrance cross section ($\delta^{**} = \delta_T^{**} = 0$ at x = 0), then the solution of the integral equations (1.1) and (1.2) for the boundary layer, together with (1.9) and (1.10), yields:

for the dynamic layer

$$\delta^{**} = 0.0366\,\mathrm{Re}_x^{-0.2} x (\cos\varphi)^{-0.6}, \quad \frac{c_{fx}}{2} = 0.029\,\mathrm{Re}_x^{-0.2}(\cos\varphi)^{-0.6}; \quad (1.11)$$

for the thermal layer with q = const

$$\delta_T^{**} = 0.0306\,\mathrm{Re}_x^{-0.2} x\,\mathrm{Pr}^{-0.6}(\cos\varphi)^{-0.6},$$
$$\mathrm{St} = 0.0306\,\mathrm{Re}_x^{-0.2}\,\mathrm{Pr}^{-0.6}(\cos\varphi)^{-0.6}. \quad (1.12)$$

As $\varphi \to 0$ these equations change into the well-known equations for unswirled flow [22].

Thus, the friction in the axial direction and the coefficient of heat and mass transfer in a swirled stream are higher than in an unswirled stream by a factor of $(\cos\phi)^{-0.6}$ at the same distance from the entrance and for the same longitudinal velocity component at the outer limit of the boundary layer, if the additional action of mass forces on turbulent transfer is not taken into account.

The so-called method of straightening of streamlines [13, 16], which essentially consists in solving the boundary-layer equations constructed for the direction along a streamline, has gained wide popularity in the analysis of boundary layers in swirled streams. A certain arbitrariness in the treatment of this method, however, leads to a difference in the final calculating equations obtained by different investigators.

Let us dwell in somewhat more detail on an analysis of the principle of the straightening of streamlines. We represent the equations of momentum and energy in an orthogonal coordinate system connected with a streamline at the surface of a thin boundary layer ($\delta/R \ll 1$) for zero-gradient flow as [21]

$$\frac{d\delta_\Sigma^{**}}{dL} = \frac{c_{f\Sigma}}{2}, \quad \frac{d\delta_{T\Sigma}^{**}\Delta T}{\Delta T\,dL} = \mathrm{St}, \quad (1.13)$$

where L is the coordinate measured along the streamline ($dL = dx/\cos\varphi$). The integral parameters in (1.13) have the form

$$\delta_\Sigma^{**} = \int_0^\delta \frac{\rho u_\Sigma}{\rho u_{\Sigma 0}}\left(1 - \frac{u_\Sigma}{u_{\Sigma 0}}\right)\left(1 - \frac{y}{R}\right)dy, \quad \delta_{T\Sigma}^{**} = \int_0^{\delta_T} \frac{\rho u_\Sigma}{\rho_0 u_{\Sigma 0}}\left(1 - \frac{T - T_w}{T_0 - T_w}\right)\left(1 - \frac{y}{R}\right)dy,$$

while the coefficients of friction and heat exchange are, respectively,

$$\frac{c_{f\Sigma}}{2} = \frac{\tau_\Sigma}{\rho_0 u_{\Sigma 0}^2}, \quad \mathrm{St}_\Sigma = \frac{-q_w}{\rho_0 u_{\Sigma 0} c_p \Delta T}.$$

As can be seen, Eqs. (1.13) are similar to the integral equations for flow without swirling [22]. Consequently, the solutions of these equations will also be similar if the criteria appearing in them are determined from the total velocity and the length of the streamline, neglecting the influence of centrifugal forces on transfer:

$$\frac{c_{f\Sigma}}{2} = \frac{B}{2}\mathrm{Re}_\Sigma^{**-m}, \quad \mathrm{St}_\Sigma = \frac{B}{2}\mathrm{Re}_{T\Sigma}^{**-m}\,\mathrm{Pr}^{m-1}. \quad (1.14)$$

If Eqs. (1.14) are transformed, using the longitudinal velocity component u_{x0} and the x coordinate as the scales, they change into Eqs. (1.9)-(1.12). This means that the method of straightening streamlines yields results similar to those obtained using the hypothesis of a three-dimensional boundary layer.

2. Influence of Mass Forces on Turbulent Transfer of Momentum and Heat in a Swirled Stream

In the analysis of the influence of mass forces on turbulent heat and mass transfer and friction in flow over curved surfaces, the method of [20], based on the analogy between the action of repulsive forces in a stratified stream and centrifugal forces at a surface with longitudinal curvature, has gained wide popularity. It is proposed to allow for the influence of curvature quantitatively in the form of empirical dependences of the linear scale of the turbulence on the Richardson number

$$l/l_0 = 1 - \beta\,\mathrm{Ri}. \tag{2.1}$$

Here β is an empirical constant; Ri is the Richardson number. The problem of choosing the empirical coefficient β, which varies within wide limits according to the data of different authors, complicates the use of the method.

Following [15], we analyze the influence of centrifugal forces on the turbulent characteristics in the boundary layer of a swirled stream. For this we assume that the influence of centrifugal forces in a swirled stream is manifested through a change in the radial pulsation velocity. The turbulent shear stresses in the longitudinal direction and the turbulent heat flux in the boundary layer, with similarity of the fields of circulation and longitudinal velocities, can be represented in the form [14, 15]

$$\tau_x = \rho l_0^2 \frac{\partial u_x}{\partial r}\left[\left(\frac{\partial u_x}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u_\varphi r}{\partial r}\right)^2\right]^{1/2} f; \tag{2.2}$$

$$q = \rho c_p l_0 l_{T0} \frac{\partial T}{\partial r}\left[\left(\frac{\partial u_x}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u_\varphi r}{\partial r}\right)^2\right]^{1/2} f. \tag{2.3}$$

The function f, allowing for the influence of the curvature of streamlines, is described by the relations

$$f = (l/l_0)^2 = \sqrt{1-(y/l_0)^2\,\mathrm{Ri}} \tag{2.4}$$

for flow at a concave surface and

$$f = (l/l_0)^2 = 1/\sqrt{1+(y/l_0)^2\,\mathrm{Ri}} \tag{2.5}$$

for flow at a convex surface. The Richardson number in (2.4) and (2.5), characterizing the ratio of the production turbulent energy by mass forces to the production of shear stresses [20], is

$$\mathrm{Ri} = \left(\frac{2u_\varphi}{r^2}\frac{\partial (ru_\varphi)}{\partial r} + \frac{1}{\rho}\frac{\partial \rho}{\partial r}\frac{u_\varphi^2}{r}\right)\Big/\left[\left|\frac{\partial u_x}{\partial r}\right|^2 + \left|\frac{1}{r}\frac{\partial (ru_\varphi)}{\partial r}\right|^2\right]. \tag{2.6}$$

As can be seen, in the general case the mass forces can be due not only to the gradient of circulation but also to the gradient of density in the boundary layer. For a constant swirling angle over the thickness of the boundary layer and similarity of the profiles of circulation and of longitudinal and total velocity,

$$u_x/u_{x0} = u_\varphi r/u_{\varphi 0}R = u_\Sigma/u_{\Sigma 0} = \omega$$

Eq. (2.6) can be reduced to the form

$$\mathrm{Ri} = \mp\frac{\delta}{R}\sin^2\varphi\left[2\omega\frac{\partial\omega}{\partial\xi} + \frac{\omega^2}{\rho}\frac{\partial\rho}{\partial\xi}\right]\Big/\left[\frac{\partial\omega}{\partial\xi}\right]^2, \tag{2.7}$$

where $\xi = y/\delta$; R is the radius of the channel; a minus sign corresponds to flow over a concave surface and a plus sign to flow over a convex surface. With allowance for the equation of state for an ideal gas, $\rho/\rho_0 = T_0/T$, and the similarity of the velocity and temperature fields, $\omega = \Theta = \xi^n$, after simple transformations we obtain from (2.7)

$$\mathrm{Ri} = \mp\frac{\delta}{R}\sin^2\varphi\left[\frac{2\xi}{n} + \frac{\xi}{n}\frac{\psi-1}{\psi/\Theta+1-\psi}\right]. \tag{2.8}$$

Since the influence of mass forces is felt predominantly through the outer part of the boundary layer, we can average the quantity Θ on the right side of (2.8) over the thickness of the layer without introducing a significant error. We write the final expression for the Richardson number in a nonisothermal boundary layer as

$$\mathrm{Ri} = \mp\frac{2\delta\xi}{Rn}\sin^2\varphi\left[1 + \frac{\psi-1}{2(\psi n+1)}\right]. \tag{2.9}$$

For isothermal flow ($\psi = 1$) Eq. (2.9) is simplified:

$$\mathrm{Ri} = \mp\frac{2\delta\xi}{Rn}\sin^2\varphi = \mp\frac{2\delta^{**}\xi(1+n)(1+2n)}{Rn^2}. \tag{2.10}$$

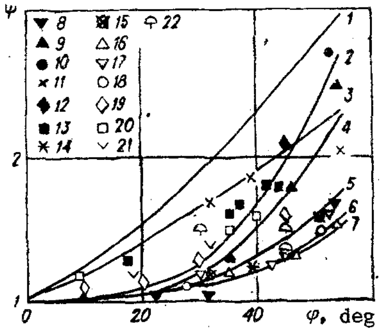

Fig. 1

Here we must keep in mind that in the general case the exponent n in the power-law velocity profile depends on the curvature of the flow and the nonisothermicity. In a first approximation, however, we can take $n = 1/7$. Thus, the integral parameter characterizing the action of curvature on friction and heat exchange, as has been pointed out in a number of papers [20, 23], is the ratio of the thickness of the boundary layer (or its integral scale δ^{**}) to the radius of curvature. The expressions (2.2)-(2.5) and (2.9) consist of a system of equations describing the distribution of turbulent shear stresses and heat fluxes, which can be used for calculations of surface friction and heat transfer. Here one need not use additional empirical constants allowing for the influence of mass forces on turbulent exchange.

From a joint solution of the above-indicated system of equations and the limiting integral of the asymptotic theory of a boundary layer [22] one can calculate, by the method of successive approximations, the influence of the curvature of streamlines on friction and heat and mass exchange. The results of a numerical calculation are approximated well by the relations for a concave wall (flow in a pipe)

$$\Psi_c=\left\{1+1.8\cdot 10^3\frac{\delta^{**}\sin^2\varphi}{R}\left[1+\frac{\psi-1}{2(\psi n+1)}\right]\right\}^{0.162}\quad \text{for } \delta^{**}/R < 0.025 \tag{2.11}$$

and for a convex wall (flow over a cylinder)

$$\Psi_c=\left\{1+2.2\cdot 10^3\frac{\delta^{**}\sin^2\varphi}{R}\left[1+\frac{\psi-1}{2(\psi n+1)}\right]\right\}^{-0.115}\quad \text{for } \delta^{**}/R < 0.01. \tag{2.12}$$

These equations are valid for calculations of turbulent heat and mass transfer with $Pr \approx 1$. In the case of $Pr > 1$ ($Sc > 1$) the factor $Pr^{-1.8}$ ($Sc^{-1.8}$) appears in (2.11) and (2.12) for δ^{**} [14].

3. Discussion of Results

A comparison of experimental and calculated data of different authors on heat exchange and friction of a fully swirled stream in a pipe is given in Fig. 1. The papers from which these data were taken, as well as the calculating equations and the experimental conditions, are given in Table 1; the light points correspond to tests on heat and mass exchange and the dark points correspond to those on friction. The calculated relations and experimental results on friction are reduced to the longitudinal component of the shear stress. The data of Fig. 1 are treated in the form of the dependence of the relative coefficient of heat and mass exchange and friction $\Psi = (c_{fx}/c_{f0})_{Re_x^{**}} = (St/St_0)_{Re_T^{**}}$ on the swirling angle for equal Re^{**} ($Re_x^{**} = \rho u_{x0}\delta_x^{**}/\mu$ and $Re_T^{**} = \rho_0 u_{x0}\delta_T^{**}/\mu$) for swirled and unswirled flows. We note that for stabilized section of flow in a pipe, the relative coefficients of friction for Re^{**} = idem and Re_D = idem coincide with each other; the analysis was made using the latter condition.

Curve 7 is a calculation from Eqs. (1.9) and (1.10), describing the intensification of friction and heat exchange ($Pr = 1$) in the absence of the influence of centrifugal forces on turbulent transfer. Line 1 reflects the combined action of an increase in the total velocity and in the mass forces in a swirled stream on transfer processes. The calculation was made for $\delta/R = 1$. Naturally, such a flow regime with stream swirling does not occur in practice. This calculated estimate characterizes the limiting influence of mass forces on friction and heat exchange, however.

As can be seen from Fig. 1, none of the existing theories describes the experimental material in full measure. All the test data lie in the interval between the lines of maximum (7) and minimum (1) action of centrifugal forces. The scatter of the experimental results

TABLE 1

Lines, points of Fig. 1	Source	Calculated or measured parameters	Calculating equation or experimental conditions
1	Present work	Friction and heat exchange with allowance for mass forces	$\Psi = \Psi_\varphi \Psi_c$
2	[13]	Friction	$\Psi = (\cos\varphi)^{-1,75}$
3	[17]	Heat exchange	$\Psi = 1+(\mathrm{tg}\,\varphi)^{-1,75}$
4	[16]	»	$\Psi = (\cos\varphi)^{-1,5}\ f(x)$
5	[18]	Friction and heat and mass exchange	$\Psi = 1/\cos\varphi$
6	[13]	Heat and mass exchange	$\Psi = (\cos\varphi)^{-0,8}$
7	Present work	Friction and heat exchange without allowance for mass forces	$\Psi = (\cos\varphi)^{-0,75}$
8	[6]	Friction	Twisted wire or ribbon
9	[8]	»	Vane swirlers
10	[10]	»	Spiral insert
11	[1]	»	Ribbon swirler
12	[4]	»	Twisted ribbon or helix
13	[9]	»	Ribbon swirler
14	[2]	»	» »
15	[5]	Heat exchange	» »
16	[8]	»	Vane »
17	[1]	»	Ribbon »
18	[3]	Mass exchange	Tangential entry
19	[4]	Heat exchange	Twisted ribbon or helix
20	[9]	»	Ribbon swirler
21	[11]	»	» »
22	[18]	»	Vane »

gives reason to think that the swirling angle does not uniquely reflect the intensification of friction and heat exchange.

In the general case, because of the different conditions of entry, as well as the specific features of the formation of the swirled stream by different means (helix, twisted ribbon, etc.), the thickness of the boundary layer in the stabilized section does not equal the radius of the channel, as occurs in an unswirled stream. Consequently, the diameter of the channel and the average flow-rate velocity in it are not determining quantities in calculations of transfer processes in swirled streams. In the comparisons available in the literature [13, 17, 18] these factors were not all taken fully into account, while the correspondence between theory and experiment noted in the papers is probably explained by mutual influences of these factors on the total coefficient of friction and heat and mass transfer. Nor was the action of mass forces on the intensification of transfer processes taken into account.

The size of the contribution of mass forces to the total coefficient of heat and mass transfer can be judged from Fig. 2, where the influence of the increase in velocity at the outer limit of the boundary layer and of centrifugal forces is isolated. The calculated and experimental functions reflecting the intensification of heat and mass transfer due to mass forces are represented by solid lines and points, respectively. The experimental data are taken from [12]; 1 and 3 reflect the intensification of mass exchange under strongly nonisothermal conditions with heating of the wall (graphite burning, $\psi = 7$, Sc = 0.7) for a practically constant swirling angle $\varphi = 32°$ along the channel length, 4 is for quasi-isothermal conditions ($\psi \approx 1$), while 2 and 5 correspond to mass exchange during the sublimation of naphthalene ($\psi = 1$, Sc = 2.6) for the same swirling angle. The calculated curves were determined from Eq. (2.11) (using Sc > 1). Line 6 is the relative coefficient of heat and mass exchange with allowance for the increase in the absolute velocity at the outer limit of the boundary layer, $\Psi_\varphi = 1/(\cos\varphi)^{0.75}$ for $\varphi = 32°$. As can be seen, for the conditions under consideration, the contribution of mass forces is comparable with the intensification of heat and mass transfer due to the increase in total velocity only for Sc = 2.6 (points 2 and line 5); for isothermal flow (line 4) and especially for heating of the wall (points 1, line 3) the influence of mass forces prevails; this fact must be taken into account in the analysis of friction and heat exchange for swirled streams.

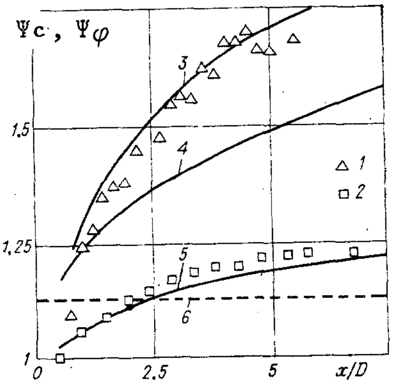

Fig. 2

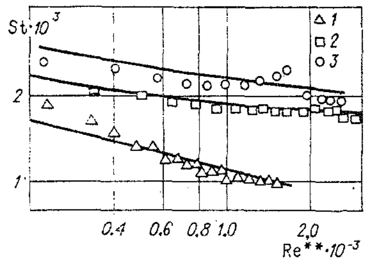

Fig. 3

The absence of detailed information in the literature on the aerodynamics of swirled streams, e.g., the distributions along the channel length of the swirling angle, the thickness of the boundary layer, and the longitudinal velocity at its limit, prevented us from making a detailed comparison of the experimental data of various investigators with calculation. For a comparison with the results of the theory, we used the data of [12] on a study of mass exchange in the initial section of a burning graphite pipe ($\psi \approx 7$). The results of the tests for $\varphi = 0$, 32, and 45° (points 1-3) are compared with calculation in Fig. 3. The calculation was made from the equation

$$\mathrm{St} = \mathrm{St}_0 \Psi_T \Psi_\varphi \Psi_c = 0.0128\, \mathrm{Re}_T^{**-0.25} \left(\frac{\mu_w}{\mu_0}\right)^{0.25} \mathrm{Sc}^{-0.75} (\cos\varphi)^{-0.75} \left(\frac{2}{\sqrt{\psi}+1}\right)^2 \times$$
$$\times \left\{1 + 1.8 \cdot 10^3 \frac{\delta^{**}}{R} \sin^2\varphi \left[1 + \frac{\psi - 1}{2(\psi n + 1)}\right]\right\}^{0.162}.$$

In this case the heat- and mass-exchange function allows for the nonisothermicity of the flow (Ψ_T), the increase in total velocity (Ψ_φ), and the intensification due to mass forces (Ψ_c). The values of δ^{**} and Re_T^{**} were found from the solution of the integral equations of motion, energy, or mass exchange; these solutions can be obtained in the same way as the functions (1.11) and (1.12):

$$\delta^{**} = 0.0306\, \mathrm{Re}_x^{-0.2}\, x (\Psi_T \Psi_\varphi \Psi_c)^{0.8},$$
$$\mathrm{Re}_T^{**} = 0.0366\, \mathrm{Re}_x^{0.8} (\Psi_T \Psi_\varphi \Psi_c)^{0.8}\, \mathrm{Pr}^{-0.6} \left(\frac{\mu_w}{\mu_0}\right)^{0.2}.$$

As can be seen, the calculation gives good agreement with experiment for such complicated conditions, with mass forces making the largest contribution to the intensification of mass exchange for this nonisothermal flow.

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HYDRODYNAMICS AND HEAT TRANSFER IN THE VOLUME OF A UNIFORM LIQUID WITH INDUCED TURBULENCE

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Many processes in chemical engineering are based on mass transfer between solid particles and a fluid. Mass transfer can be intensified by producing an organized average motion of the fluid or by agitating the latter. An increase in the level of turbulence in the fluid is an effective means of increasing the mass-transfer coefficient. The problem of determining the flow of a highly turbulent flow about a particle is extremely complex, and the literature presently contains very little information on the mechanism of turbulent transport and, in general, the effect of turbulence on transport processes.

One of the simplest methods of developing a high turbulence intensity is agitating the fluid in a vessel with an oscillating grid. Several studies have examined the fluid dynamics of such a flow. In these investigations, the frequency of oscillation of the grid ranged within f = 1-6 Hz. It was shown in [1] that after the grid is induced to move in the measurement volume — located a certain distance from the grid — the RMS fluctuations of fluid velocity

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